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Report

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Abstract

We demonstrate here that the origin of the Ramdas layer or the lifted temperature minimum (LTM), first observed by Ramdas and co-workers in the 1930's ([21]) still remains a mystery. A recent theory ([2]) that purports to explain the preferential cooling of the near-surface air layers leading to the LTM, based on radiative transfer processes in a homogeneous water-vapor-laden atmosphere, is shown to be fundamentally inconsistent. The exaggerated effect of the reduced ground emissivity on the near-surface cooling rates (infrared flux divergences) predicted by the theory is spurious, and is due to a physically incorrect 'band cross-talk'. The error arises in the treatment of the reflected radiation. This is shown by comparing the flux divergences obtained using gray and band model formulations for the simplistic case of a participating medium with only two bands. It is then argued that the cross-talk error in varying amounts is, in fact, inherent in a naive radiative heat transfer model, involving non-black emitting surfaces, that does not fully resolve the emission spectrum of the participating medium. For the Ramdas layer in particular, the discrepancy between a naive gray model and more accurate band models is greatly magnified due to the rapidly fluctuating nature of the emission spectrum of the principal radiating component (water vapor). It is finally shown that a careful treatment of the reflection term, even within the purview of a gray theory, eliminates the aforementioned spurious cooling. The inevitable conclusion from our analysis is that radiative processes, acting in a homogeneous isothermal atmosphere, will not lead to a preferential cooling of the air layers near the ground, and thence, to an LTM. The origin of the LTM must therefore lie in an atmosphere that is heterogeneous on the same length scales. We discuss the role of aerosols as a likely candidate for this heterogeneity.

1 Introduction

This report focusses on the origin of a well-known micro-meteorological paradox variously termed as the Ramdas layer, the lifted temperature minimum (LTM), the elevated minimum etc ([21];[23]; [13]). Originally observed by Ramdas and co-workers in the 1930's, the Ramdas paradox concerns the occurrence of a minimum in the night-time temperature profile a few tens of centimeters above the ground on calm clear nights. The phenomenon was initially thought to be restricted to the tropics; however, it has since been shown to be quite robust, having been observed all over the world over varied surfaces including rough soil, bare soil, Aluminum and concrete surfaces etc. under calm cloudless conditions ([1]; [4]; [12]; [19]; [22]; [15]). Despite its robustness, the phenomenon remains counterintuitive in more ways than one. First, solid surfaces are far more effective radiators compared to gases. Efficient absorption of short-wave radiation leads to the ground heating up rapidly during the day, and likewise, emission in the infra-red range to outer space must lead to a rapid cooling at night. Thus, conventional wisdom based on a nocturnal inversion layer suggests that ground, and not the air layers above it, must be the coldest at night ([27]). Figure 1 shows a schematic of the familiar day-time and night-time temperature profiles together with a night-time temperature profile exhibiting an LTM. Secondly, as already pointed out earlier ([16]), the relevance of a length scale of the order of decimeters in an atmosphere that is apparently homogeneous on length scales of the order of a kilometer is far from obvious. Thirdly, the Rayleigh number calculated based on the height and intensity of the LTM is $O(10^6)$, a factor of 1000 greater than the familiar critical threshold one would estimate based on the usual balance of conduction and convection ([6]). The apparent persistence of the LTM through the night, with no perceptible indication of an overturning instability, is again perplexing. It is thought that the stabilizing effects of cooling due to long-wave radiation may play a role, although there exist no quantitative predictions in this regard. Assuming a homogeneous atmosphere [1], however, the most

[1] this is the central assumption in the VSN model to be discussed later; the only relevant length scale in the theory, indicative of an inhomogeneous atmosphere, is the water vapor scale height. The latter

puzzling aspect of the LTM, is the existence of a cold layer of air in the immediate neighborhood of warmer ground and warmer layers of air above. Since any cooling below the ground temperature can only arise on account of radiation to outer space, the occurrence of the LTM implies that layers of air closest to the ground radiate most efficiently to outer space (here, we use the term 'outer space' to denote the upper reaches of the atmosphere where the temperature, on account of the adiabatic lapse rate, $\sim 10Kkm^{-1}$, has dropped to a level significantly below the ground temperature). This is difficult to explain since radiant heat exchange, although a strongly non-local process in certain regions of the infra-red spectrum, is still a decaying function of the distance between participating elements in a homogeneous medium; for instance, the transmissivity function monotonically decreases with the height of a homogeneous water-vapor-laden air column.

Thus, initial observations of the LTM were variously attributed to instrumentation error, advection of horizontal inhomogeneities (cold air from the environs), etc; however, more definitive experiments by Raschke ([22]), and very recently by Mukund and Sreenivas ([15]), have established that radiation plays a key role in the phenomenon. The Ramdas layer highlights in general the important but subtle role played by radiative processes in the stable atmospheric surface layer. The need to accurately resolve the temperature variation in the lowest meters of the atmosphere is also of particular importance in agricultural meteorology (the occurrence of frost and its adverse effect on crops ([12])) and remote sensing (determination of true surface temperatures and emissivities ([5]))

The presently accepted theoretical explanation for the origin of the LTM is the VSN model that was proposed in 1993 ([2]). More detailed accounts of various aspects of the original theoretical formulation, and with the inclusion of additional factors such as turbulence, have since appeared elsewhere in a series of papers ([17]; [18]; [20]; [3]). The theory is essentially an application of the well-known flux-emissivity model for radiative heat transfer ([14]) to a water-vapor-laden atmosphere. The flux-emissivity is determined

is about $3km$, a factor of $O(10^4)$ greater than the length scales relevant to the LTM, and therefore, irrelevant to its existence.

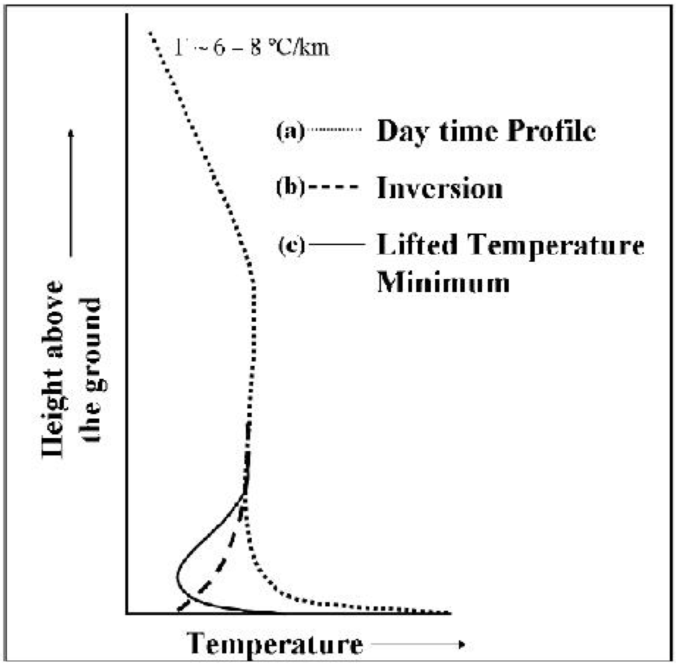

Figure 1: The figure shows three temperature profiles typically encountered in the lowest meters of the atmosphere. The dotted and dashed lines denote the expected day-time and night-time temperature profiles, the latter characteristic of a near-surface inversion layer. The solid line denotes the non-monotonic variation of temperature very near the ground leading to the emergence of an elevated minimum.

from an experimental fit down to length scales (tens of centimeters) relevant to the LTM. Unlike earlier theories ([32]) which invoked the presence of a haze layer near the ground in order to explain the LTM, the VSN model predicts a minimum in temperature at a height of a few tens of centimeters in a homogeneous night-time atmosphere. The physics underlying the minimum has been attributed to the smoothing of a radiative slip by conduction; the slip itself leads to the air being substantially cooler than the ground in a near-surface, emissivity sub-layer, with a height of the order of a few meters (of the order of the photon mean-free-path in the strongly absorbing bands of the water vapor spectrum). Further, such a sub-layer is crucially predicted to occur only for a non-black surface. Herein, we show that both the cooling in the emissivity sub-layer and the resulting temperature minimum in presence of conduction are spurious effects resulting from band cross-talk, that is, a physically incorrect coupling between the most opaque and most transparent bands in the water vapor emission-spectrum. The emissivity sub-layer, in the context of the VSN model, is merely a boundary layer where the spurious cooling caused by the cross-talk term is the most significant. The issue of band cross-talk has implications beyond the specific context of the Ramdas layer, and highlights the need for a more careful treatment of the reflected radiation within the framework of traditional flux-emissivity models for radiative heat transfer ([26]). Indeed, the VSN model is not the first model to suffer from an erroneous expression for the reflected flux. However, the model is certainly an instance where the spurious effects of band cross-talk have been most exaggerated, and been attributed as the sole reason for the occurrence of a physical phenomenon. As will be shown later in section 2, this happens due to the detailed spatial resolution of the flux-emissivity variation, without a commensurate spectral resolution, within an erroneous gray flux-emissivity formulation. The magnified cross-talk leads to a cooling rate in the lowest layers of air of about $1700 K day^{-1}$ for a ground emissivity of $\epsilon_g = 0.8$. This cooling rate decays over a length scale of the order of meters, the photon path-length of the strongly absorbing bands. Removing the erroneous cross-talk term leads to a more familiar but considerably more modest cooling rate of about $5-6 K day^{-1}$ that is now spread out over a length scale of the order of kilometers for both black and non-black surfaces ([26]). Further, the cooling rate now monotonically increases with height consistent with the intuitive notion that the upper air layers radiate more effectively to outer space than the layers below.

The report is organized as follows. In the next section, we first introduce the flux-emissivity formulation for a participating medium in a plane-parallel atmosphere. The one-dimensional plane-parallel context is no restriction since, as discussed in detail by Vasudevamurthy (1986), earlier experiments on the LTM have conclusively ruled out the role of advection of horizontal inhomogeneities. In order to clarify the origin of band cross-talk in the VSN model, we consider a homogeneous isothermal atmosphere whose emission spectrum consists of only two bands - a non-window band with a path length much shorter than the height of the atmosphere, and a window band where the path length is comparable to or greater than the height of the atmosphere. The two-band scenario, although simplistic, captures the essence of the wildly fluctuating nature of the water vapor emission spectrum in the infra-red range, at least as far as its role in the LTM is concerned. In this region of the spectrum, the optical path length may vary from a few meters in the strongly absorbing bands to the order of kilometers in the nearly transparent window band $(8-12\mu)$. The expressions for the radiative flux divergence are first obtained using a gray flux-emissivity formulation that averages the interaction in the two bands, and then, using a two-band formulation where the flux-emissivity is defined separately in each of the two bands. It is shown that, for a non-black surface, the gray formulation leads to additional physically incorrect terms that denote the rapid attenuation of the missing energy in the reflected radiation, in the transparent window band, by the non-window absorption coefficient. This term leads to a strong but spurious cooling of the near-surface layers of air. The discrepancy between the gray and band formulations is related to the standard deviation of the emission spectrum as calculated from the band model. The latter quantity is quite large, since, unlike solids, the absorption coefficient of atmospheric gases like water vapor and carbon dioxide is a rapidly fluctuating function of wavelength. A later band model used by the same authors ([28]), in fact, does not produce an LTM. Although closer to the correct answer (of there being no LTM in a homogeneous atmosphere), the band model still suffers from a cross-talk error (between sub-bands of differing opacity) that leads to a rather muted temperature minimum at a height of several meters; more importantly, the band model continues to predict an erroneous cooling-rate profile that peaks near the ground. The error committed highlights the need for an educated choice and implementation of band-models. It is finally shown that a careful treatment of the reflected radiation, even within a gray formulation, that accounts for the different path-lengths of the emitted and reflected components of the upwelling radiant flux, eliminates the spurious band cross-talk. Having clarified the origin of the error in the VSN model, we return to a discussion of the origin of the LTM in section 3. Evidence is presented that shows the LTM to not be a remnant of the day-time temperature profile.

Further, the evolution of the temperature profile after a turbulence episode confirms that, although not at the same temperature to begin with, the near-surface layers of air cool faster than the ground. Thus, the origin of the LTM may indeed be attributed to an intrinsic radiative cooling mechanism of the near-surface layers of air. The immediate implication is that the near-surface layers of air must be different from those above, and the presence of an LTM must therefore reflect an atmosphere that is inhomogeneous on the relevant length scales (a few decimeters). The available data, although admittedly inadequate, appears to point toward the presence of a greater concentration of aerosols in the lowest layers of air as a plausible candidate for this inhomogeneity. The mechanism of cooling on account of such a heterogeneity is discussed again within the context of a flux-emissivity formulation.

2 Calculation of radiative flux divergences using the flux-emissivity model: the origin of band cross-talk

The flux-emissivity model for radiative heat transfer is explained in detail in standard textbooks ([14]), and we directly write down the expressions for the four distinct contributions to the net radiative flux $F(u)$ as a function of u. Here, u is a mass absorber path length, and for an inhomogeneous atmosphere with water vapor as the principal radiating component, is given by $u(z) = \int_0^z \rho_w(z')\left(\frac{p(z')}{p(0)}\right)^{\delta} dz'$, where $\rho_w(z)$ and $p(z)$ are the water vapor density and the pressure, respectively, at a height z, and δ is an empirical constant typically in the range $0.5 < \delta < 1$ ([11]). The scale height for water vapor is about $2.7km$, while the pressure scale height is still greater being about $8km$. For the length scales of interest, the atmosphere may therefore be regarded as homogeneous, and we will treat $u\,(\approx \rho_w(0)z)$ as a proxy height; with a typical value of $\rho_w \approx 0.01kg/m^3$, an actual height of $1km$ corresponds to $u \approx 10kg/m^2$. Thus, within a gray flux-emissivity formulation,

$$F(u) = F_{eg}^{\uparrow}(u) + F_a^{\uparrow}(u) + F_{rg}^{\uparrow}(u) - F_a^{\downarrow}(u), \tag{1}$$

where $F_{eg}^{\uparrow}(u)$ denotes emission from the ground, $F_a^{\uparrow}(u)$ and $F_a^{\downarrow}(u)$, respectively, denote the upward and downward emissions of the air layers above and below the reference level u, and $F_{rg}^{\uparrow}(u)$ denotes the upwelling reflected flux from ground that is not perfectly black. Taking the ground at $u = 0$, and the top of the atmosphere at $u = u_\infty$, the expressions for the individual flux contributions are given by

$$\begin{aligned}
F_{eg}^{\uparrow}(u) &= \epsilon_g \sigma T_g^4 (1 - \epsilon(u)), & (2)\\
F_a^{\uparrow}(u) &= -\int_0^u \sigma T^4(u', t)\dot{\epsilon}(u - u')du', & (3)\\
F_a^{\downarrow}(u) &= \int_{u_\infty}^u \sigma T^4(u', t)\dot{\epsilon}(u' - u)du', & (4)\\
F_{rg}^{\uparrow}(u) &= (1 - \epsilon_g)(1 - \epsilon(u))F_a^{\downarrow}(0), & (5)
\end{aligned}$$

where $T(u,t)$ denotes the air temperature at a height u and at time t, and $\dot{\epsilon}(u) = d\epsilon/du$. Here, T_g is the ground temperature, and ϵ_g denotes the ground emissivity assumed to be independent of wavelength (frequency). Of course, the central quantity in the above formulation, the flux-emissivity $\epsilon(u)$, may be interpreted as the emissivity of an atmospheric column of height u. The Janus-faced nature of the water vapor emission-spectrum is captured herein by the following simple expression for the gray flux-emissivity:

$$\epsilon(u) = 1 - f_w \mathrm{e}^{-\alpha_w u} - f_{nw} \mathrm{e}^{-\alpha_{nw} u}, \tag{6}$$

valid for a two-band emission spectrum. Here, the subscripts 'w' and 'nw', respectively, denote the transparent window band and the strongly absorbing non-window band. The corresponding absorption coefficients are α_w and α_{nw} with the fractions of the total radiant energy in the window and non-window bands being f_w and f_{nw}; thus, $f_w + f_{nw} = 1$. The above expression may be obtained from the exact expression for the gray flux-emissivity by assuming a constant absorption coefficient in each of the two bands, and approximating the resulting exponential damping of the radiant intensity along any given direction by an angularly averaged exponential in the vertical coordinate. Thus, the inverses of the absorption coefficients in the individual bands are to be interpreted as angularly averaged photon mean-free-paths. The length scales in the problem are ordered as $\alpha_{nw}^{-1} \ll u_\infty \ll \alpha_w^{-1}$; in other words, the atmosphere is transparent in the window band, while being nearly opaque in the non-window band. Now, the flux emissivity in the VSN model is determined from an empirical fit given in terms of a logarithmic function of u ([2]). However, the above linear combination of exponentials retains the most important characteristic of the VSN flux-emissivity; the initial steep increase to a value of $(1 - f_w)$ over a length scale of $O(\alpha_{nw}^{-1})$ on account of the strongly absorbing bands, and a much slower increase thereafter to unity, over a length scale of $O(\alpha_{nw}^{-1})$, indicative of a saturation of the opaque bands and subsequent weak absorption in the window band ([16]). Saturation of the latter, of course, doesn't happen over an atmosphere of height u_∞, since $u_\infty \ll \alpha_w^{-1}$. Thus, in the domain $(0, u_\infty)$, to a good approximation, $\epsilon(u) \approx (1 - f_w)(1 - \mathrm{e}^{-\alpha_{nw} u})$.

The prediction of an LTM by the VSN model hinges on the generation of a radiative slip $(T_g - T(0^+, t) > 0)$ in an initially homogeneous isothermal atmosphere ([3]). The slip is the result of a radiative flux divergence profile that peaks at the ground and decays thereafter over a length scale of the order of meters. The latter length scale corresponds to the photon mean-free-path in the strongly absorbing bands of the water vapor spectrum $(\alpha_{nw}^{-1}$ in our notation). The resulting layer of cold air immediately above warmer ground has been termed the emissivity sub-layer. With the inclusion of conduction, this discontinuity in temperature is smoothed into a boundary layer that now exhibits a non-monotonic temperature dependence - the LTM (see figure 2). The initial radiative slip, due to a preferential cooling of the near-surface air layers in an isothermal homogeneous atmosphere, is thus crucial to the existence of the LTM. We will therefore focus on the radiative flux divergence in an isothermal atmosphere at a temperature T_0. In this limit,

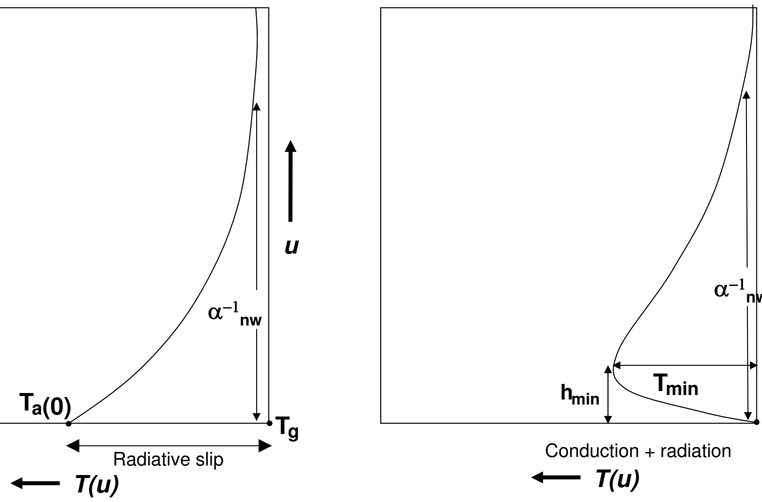

Figure 2: The temperature profiles predicted by the VSN model. In the absence of conduction, the model predicts a layer of cold air of height α_{nw}^{-1} (in the present notation) with an associated slip at the ground; conduction smoothens the radiative slip into a temperature profile exhibiting an LTM.

the flux-emissivity has a more intuitive interpretation, since the radiant flux emitted by a column of air of height u now takes the familiar form given by the Stefan-Boltzmann law viz. $\epsilon(u)\sigma T_0^4$. Using (6) for $\epsilon(u)$, the expressions for the various flux divergence contributions take the form:

$$\frac{dF_{eg}^{\uparrow}}{du} = -\epsilon_g \sigma T_g^4 [f_w \alpha_w \mathrm{e}^{-\alpha_w u} + f_{nw} \alpha_{nw} \mathrm{e}^{-\alpha_{nw} u}], \tag{7}$$

$$\frac{dF_{a}^{\uparrow}}{du} = \sigma T_0^4 [f_w \alpha_w \mathrm{e}^{-\alpha_w u} + f_{nw} \alpha_{nw} \mathrm{e}^{-\alpha_{nw} u}], \tag{8}$$

$$\frac{dF_{a}^{\downarrow}}{du} = \sigma T_0^4 [f_w \alpha_w \mathrm{e}^{-\alpha_w (u_\infty - u)} + f_{nw} \alpha_{nw} \mathrm{e}^{-\alpha_{nw} (u_\infty - u)}], \tag{9}$$

$$\frac{dF_{rg}^{\uparrow}}{du} = -(1-\epsilon_g)\epsilon(u_\infty) \sigma T_0^4 [f_w \alpha_w \mathrm{e}^{-\alpha_w u} + f_{nw} \alpha_{nw} \mathrm{e}^{-\alpha_{nw} u}], \tag{10}$$

where a negative sign denotes a heating contribution and vice-versa. The net radiative flux divergence is given by the sum of the individual (signed) contributions, and may be written as

$$\frac{dF}{du} = \sigma T_0^4\Big[-\epsilon_g(f_w\alpha_w \mathrm{e}^{-\alpha_w u}+f_{nw}\alpha_{nw}\mathrm{e}^{-\alpha_{nw}u})(K-1)+(1-\epsilon_g)(f_w\alpha_w\mathrm{e}^{-\alpha_w u}+f_{nw}\alpha_{nw}\mathrm{e}^{-\alpha_{nw}u})f_w\mathrm{e}^{-\alpha_w u_\infty} + f_w\alpha_w\mathrm{e}^{-\alpha_w(u_\infty-u)}\Big], \tag{11}$$

where $K = \frac{T_g}{T_0}$, and we have assumed $\mathrm{e}^{-\alpha_{nw}u_\infty} \approx 0$, so that $\epsilon(u_\infty) \approx (1-f_w)$. The first term in (11) represents the familiar strong cooling (when $T_g < T_0$), via opaque bands, that arises due to a difference between the ground temperature and temperature of the lowest air layer. The magnitude of this contribution is proportional to ϵ_g, being weaker for a non-black surface, and therefore cannot explain the LTM. The final term (11) denotes the 'cooling to space' contribution, and arises on account of the transparency of the atmosphere in the window band. As expected, this contribution, being proportional to α_w, predicts a weak cooling, and a flux divergence that increases with height over a length scale of $O(\alpha_w^{-1})$; in other words, the upper layers of air cool to space more efficiently. In the absence of a temperature discontinuity at the ground, that is, with $K = 1$, the dominant contribution to the cooling in the near-surface layers of air ($u \ll u_\infty$) comes from the second term. If we neglect the much smaller contribution proportional to α_w, this cooling primarily arises due to the attenuation of the missing window-band-energy in the reflected radiation, $(1-\epsilon_g)f_w\mathrm{e}^{-\alpha_w u_\infty}$, by the transmissivity in the non-window band ($\mathrm{e}^{-\alpha_{nw}u}$). In contrast to the cooling-to-space contribution, the cooling rate is now largest near the ground, being proportional to α_{nw}, and *decays* over a much shorter length scale of $O(\alpha_{nw}^{-1})$. In addition, this near-surface cooling relies crucially on a non-black surface and a transparent atmosphere ($\epsilon(u_\infty) < 1$). But, the predicted cooling is spurious; there can be no such inter-band interaction, since the window and non-window bands correspond to mutually exclusive wavelength intervals. Simply put, a photon of a given frequency cannot trigger a transition (in the infra-red range of interest, a vibration or a vibration-rotation transition) at a rate commensurate with the absorption coefficient in a different frequency interval. That such a coupling between different bands, a 'band cross-talk', is erroneous is immediately evident from the more fundamental spectral form of the flux-emissivity formulation ([14]), wherein the radiant energy balance is carried out separately at each wavelength (or frequency). In the context of the above simplistic analysis, however, the inconsistency in the above gray formulation is more easily seen by comparing its results with a two-band formulation which naturally avoids the aforementioned cross-talk between the bands. In the band formulation, one defines flux-emissivities separately in the two bands as

$$\epsilon_w(u) = (1 - \mathrm{e}^{-\alpha_w u}), \tag{12}$$

$$\epsilon_{nw}(u) = (1 - \mathrm{e}^{-\alpha_{nw} u}). \tag{13}$$

In what follows we will assume $K = 1$, so attention may be directed towards the term in (11) involving a band cross-talk. The calculation of the flux divergences, using the above flux-emissivities, proceeds in an exactly analogous fashion, and one obtains

$$\frac{dF_w}{du} = f_w(\sigma T_0^4)[\alpha_w \mathrm{e}^{-\alpha_w u}(1-\epsilon_g)\mathrm{e}^{-\alpha_w u_\infty} + \alpha_w \mathrm{e}^{-\alpha_w(u_\infty - u)}], \tag{14}$$

$$\frac{dF_{nw}}{du} = f_{nw}(\sigma T_0^4)[\alpha_{nw} \mathrm{e}^{-\alpha_{nw} u}(1-\epsilon_g)\mathrm{e}^{-\alpha_{nw} u_\infty} + \alpha_{nw} \mathrm{e}^{-\alpha_{nw}(u_\infty - u)}]. \tag{15}$$

for the flux divergences in the two bands. The total flux divergence is

$$\begin{aligned}\frac{dF}{du} &= \frac{dF_w}{du} + \frac{dF_{nw}}{du}, \\ &= \sigma T_0^4[(f_w\alpha_w \mathrm{e}^{-\alpha_w(u+u_\infty)} + f_{nw}\alpha_{nw}\mathrm{e}^{-\alpha_{nw}(u+u_\infty)})(1-\epsilon_g) + (f_w\alpha_w\mathrm{e}^{-\alpha_w(u_\infty-u)} \\ &\quad + f_{nw}\alpha_{nw}\mathrm{e}^{-\alpha_{nw}(u_\infty-u)}].\end{aligned} \tag{16}$$

As expected, there is no band cross-talk, and thence, no preferential cooling of air layers near the ground. The terms proportional to $(1-\epsilon_g)$ in both (14) and (15) remain smaller than the respective cooling-to-space contributions, and the flux divergence profiles do not differ qualitatively for black and non-black surfaces (as is the case in the VSN model).

In either case, the cooling-rate profile exhibits a weak *increase* with height over a scale of kilometers. The strong interaction present in the gray theory is now absent even in the non-window band, since any cooling on short length scales is weakened by the opacity of the band. Indeed, since $e^{-\alpha_{nw} u_\infty} \approx 0$, in general, the near-surface air layers in opaque bands can never cool to outer space; all radiative exchanges in such a band are restricted to the immediate nearly isothermal neighborhood of a given layer. The erroneous term in the gray theory implies that the ratio of the cooling rates calculated near the ground from the two formulations ((11) with $K = 1$ and (16)) is $O[(1-\epsilon_g) f_{nw} \alpha_{nw}/\alpha_w]$. Since the photon mean-free-path varies from a few meters in the opaque bands to the order of kilometers in the window band, and $(1-\epsilon_g) f_{nw} \leq 0.1$, one expects this ratio to be about a 100. It is then no surprise that the erroneous gray model goes from predicting negligible cooling for $\epsilon_g = 1$ to an exaggerated cooling rate of about $1700 K day^{-1}$ for $\epsilon_g = 0.8$. The correct band model prediction, on the other hand, is only about $5 - 6 K day^{-1}$ and varies much less with a change in ϵ_g.

The error in the gray theory may also be understood from the original expressions for the radiant fluxes viz. (2)-(5). Since the window-band emissivity of the atmosphere of height u_∞ is negligible, that is, $(1 - e^{-\alpha_w u_\infty}) \ll 1$, the downward radiant flux incident at the surface, $F_a^\downarrow(0)$, consists largely of radiation in the opaque bands, and must, on reflection, be attenuated completely on a length scale much shorter than the height of the atmosphere. On the other hand, according to (5), a significant fraction, $(1-\epsilon_g)(1-\epsilon(u_\infty))$, of the reflected radiation escapes the top of the atmosphere (in the VSN model, $\epsilon(u_\infty) \approx 0.44$ ([3])). While the fraction $(1-\epsilon(u_\infty)) \approx f_w$ relates to the transparency in the window band, the spectral content of $F^\downarrow(0)$ implies that the transparency with regard to the reflected radiation must be that corresponding to the non-window band ($\approx f_{nw} e^{-\alpha_{nw} u_\infty}$). Thus, it appears as if the reflection interaction, in the above gray formulation, changes the spectrum from being initially restricted to the opaque bands, as in $F_a^\downarrow(0)$, to having a significant fraction of its energy in the window band in the up-welling reflected radiation. This is, of course, incorrect since the reflection interaction must leave the spectrum of the incident radiation unaltered. Indeed, it is the apparent unavailability of this warming reflected component that leads to the preferential cooling near the ground.

It must be clear now that the error arises only in the treatment of the reflected radiation. This is because the reflected term is the only term that involves a 'quadratic emissivity interaction', evident from the product $\epsilon(u_\infty)\dot{\epsilon}(u)$ in (5). In the gray theory, this is approximated by the product of the two averages. On the other hand, the total flux divergence calculated from a spectral theory, by summing the contributions at each wavelength, would involve instead an average of the product, at each wavelength, of the emissivity and its derivative; that the heights at which the two quantities are evaluated are not always the same (being u and u_∞) is a matter of detail at the level of this discussion. Thus, the discrepancy in the reflected flux obtained from a gray calculation and from a fully resolved spectral calculation is related to the difference between the product of the averaged emissivities $(\langle\epsilon_\lambda\rangle^2)$ and the average of the emissivity products $(\langle\epsilon_\lambda^2\rangle)$ [2]. This is the variance of the emission/absorption spectrum, $\langle(\epsilon_\lambda - \langle\epsilon_\lambda\rangle)^2\rangle$, now interpreted as a probability distribution with ϵ_λ being treated as a random variable, and λ varying over the entire infra-red range. It is common knowledge that gases (of which water vapor is just one example considered here), unlike solids, tend to have a very spiky absorption spectrum ([25]). The resulting rapid fluctuation of the absorption coefficient even over a small wavelength interval implies that the variance of the emissivity spectrum is large, and the gray approximation in (5) is a bad one. To sum up, the underestimate of the emissivity variance in the gray theory leads to a corresponding underestimate of the warming reflected flux contribution. This deficit in the interaction of the up-welling reflected flux then leads to the spurious cooling of the near-surface air layers.

The extent of cross-talk is expected to decrease with an increasing resolution of the emission spectrum of the participating medium due to a compartmentalization of energy

[2]The averaging is defined with respect to a spectral interval, and the derivative of the flux-emissivity is with respect to u. Since u and λ are independent variables, the differentiation and averaging operations commute. Thus, $\frac{d}{du}\langle\epsilon(u)\epsilon(u_\infty) = \langle\dot{\epsilon}(u)\epsilon(u_\infty)\rangle$, and one may directly interpret the discrepancy in terms of the variance of the emissivity spectrum.

among bands. In fact, a later band model used by the same group to analyze the LTM yields near-surface cooling rates that are orders of magnitude smaller in accordance with the above expectation ([28]). The detailed band model predicts a modest cooling rate of about $40Kday^{-1}$ at the ground for $\epsilon_g = 0.8$ in contrast to the VSN model's prediction of $1700Kday^{-1}$; however, the authors wrongly attribute this discrepancy to the presence of an 'implicit slip' in the original VSN model. The band model used by the authors is one formulated by Chou *et al* (1993) and resolves the water vapor emission spectrum at two levels. The spectrum is first resolved into wavelength intervals, the 'bands', within which the Planck-function may be regarded as a constant. An additional level of resolution is, however, necessary because the absorption coefficient is a far more sensitive function of wavelength than the Planck-function. This is accomplished by dividing each band into 'sub-bands' within which the absorption coefficient is a constant. The sub-bands within a band merely serve to resolve the small-scale variation of the absorption coefficient, and need not correspond to contiguous wavelength intervals. Within a 'gray' sub-band, the flux-emissivity may be approximated by an angularly averaged exponential. Thereafter, one may define a band-averaged emissivity as a weighted average of such exponentials, in exactly the same manner as (6), where the weighting fractions now denote the fractional energy contents of the various sub-bands within a band. A gray flux-emissivity may similarly be defined as a weighted average of the band-emissivities. Interestingly, the smallest length scales resolved via Chou's formulation, corresponding to photon mean-free-paths in the most opaque sub-bands, are in the range of millimeters! It turns out that the derivative of the gray flux-emissivity at the surface, $\epsilon'(0)$, obtained using Chou's model, is greater than that used in the VSN model ($\epsilon'_{Chou}(0) \approx 106m^2/kg$, $\epsilon'_{VSN}(0) \approx 64m^2/kg$). Clearly, if the physics in the original gray formulation were correct, the band formulation would predict a cooling rate in excess of the VSN prediction. This is, of course, not the case, since, as already seen, cross-talk between bands of differing opacities is naturally absent in a band formulation.

The earlier comparison between the gray and band formulations leads one to expect a cross-talk error even within a band-level formulation if there is a significant variation in the opacity within a given band. The latter is precisely the case, and the error arises in the implementation of Chou's model by Varghese *et al* ([28]). An examination of the band parameters shows that the opacities of sub-bands within a single band differ by $O(10^6)$, corresponding to a variation in the photon mean-free-path from millimeters to kilometers as one moves from the most opaque to the most transparent sub-band within a single band. Although closer to being the correct answer (in relation to the VSN model), the band-level formulation of Varghese *et al* (2003a) is again in error on account of cross-talk among sub-bands of differing opacities. This is, in fact, highlighted by the cooling-rate profile given in figure 3 of their paper ([29]) which erroneously predicts a peak at the ground for $\epsilon_g = 0.8$, and an associated length scale of the order of meters. It is not so much the error in the magnitude ($40 K day^{-1}$) as that in the variation in the cooling-rate with height that is crucial.

We discuss the origin of the above inconsistency in some detail since, on one hand, it highlights the error resulting from the naive use of a band-averaged quantity even in a sub-band level implementation in an apparent attempt to economize on computational expenses (a later band implementation by Savijarvi (2006) ([24]) also appears to suffer from the same error). On the other hand, the implementation error in the band model again serves to reinforce the correct answer - *that radiative cooling/heating-rates, at any given altitude in the atmosphere, cannot vary on length scales much smaller than those characterizing the atmospheric variables (temperature, pressure, humidity etc.) at the same altitude*. This latter conclusion is of particular importance since it enables one, based solely on knowledge of the atmospheric model (MLS, $T1$, etc), to immediately detect signatures of cross-talk in the calculated radiative flux divergences. The wider implications of our analysis, and the recent experimental findings of Mukund and Sreenivas (2009) ([15]), in the context of the temperature distribution in the atmospheric surface layer, have been discussed elsewhere ([26]). Returning to the band model of Varghese *et al* (2003a) ([28]), the expressions for the flux contributions that enter the radiative balance within a sub-band are given (in a slightly modified notation) by

$$
\begin{aligned}
F^{\uparrow}_{eg(ji)} &= \epsilon_g \pi B_j(T_g) c^j_i \tau^j_i(u), && (17)\\
F^{\uparrow}_{a(ji)} &= c^j_i k^i_j \int_0^{u_j} [\pi B_j(T')] \tau^i_j(u_j - u'_j) du'_j, && (18)\\
F^{\downarrow}_{a(ji)} &= c^j_i k^i_j \int_{u_j}^{u_\infty} [\pi B_j(T')] \tau^i_j(u'_j - u_j) du'_j && (19)\\
F^{\uparrow}_{rg(ji)} &= (1-\epsilon_g) F^{\downarrow}_j(0) c^j_i \tau^j_i(u), && (20)
\end{aligned}
$$

where $\tau^i_j(u) = 1 - \epsilon^i_j(u)$ is the sub-band transmissivity, and the coefficients c^j_i denote the fractional energy contents of the sub-bands; here, i and j denote the sub-band and band indices, respectively. The error arises in replacing the actual down-welling flux at the ground, $F^{\downarrow}_{ji}(0)$, by an approximation based on the band-averaged value, $c^j_i F^{\downarrow}_j(0)$; writing down the expressions for the reflected flux in terms of τ^i_j leads to terms involving products of sub-band transmissivities, and therefore a sub-band cross-talk. The approximation based on the band-average flux results in an overestimation of the actual down-welling flux in the most transparent sub-bands, but underestimates the down-welling flux in the most opaque sub-bands wherein the atmosphere acts as a black body. On account of the resulting deficiency in the reflected fluxes in the opaque sub-bands, there is again a spurious cooling of the near-surface air layers on shorter length scales.

The smallness of the cross-talk error in a band-level formulation in relation to a gray formulation isn't obvious. Although the energy content is now compartmentalized among the bands, the range of opacities resolved via sub-bands is usually greater than that in the gray formulation. However, the choice of the sub-band weighting coefficients in Chou's formulation does lead to a muted temperature minimum, since the energy content of the opaque sub-bands is vanishingly small. The extent of cross-talk in any band model may be gauged from the plot of the band-averaged emissivities versus u. In figure 3, we reproduce the comparison of the band-averaged emissivities and the gray flux-emissivity used in the VSN model given in Varghese (2003a) ([28]). Crucially, in contrast to the VSN

flux-emissivity, the band-averaged emissivities that exhibit the shortest length scales are also the ones that saturate to unity due to band opacity. The discrepancy between the

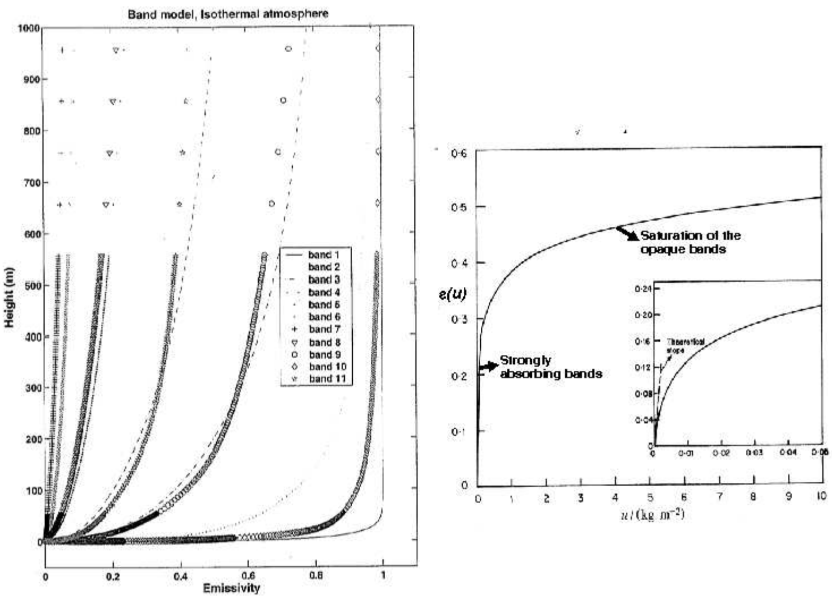

Figure 3: The figure, reproduced from Varghese (2003a), contrasts the band-averaged emissivities calculated using Chou's formulation ([7]) and the flux emissivity used in the VSN model ([2]).

gray and band formulations on account of cross-talk discussed in detail above is clearly more general than the specific instance of the Ramdas layer. Indeed, the VSN model is not the first to suffer from this error. The expressions for the radiant fluxes used in the VSN model may be traced to an earlier flux-emissivity formulation by Garrat and Brost (1981) ([10]), which also suffers from an erroneous treatment of the reflection term. However, the errors incurred in extrapolating cooling rate predictions to a gray surface are much milder in comparison: for instance, the cooling rate increased from $17Kday^{-1}$ to $72Kday^{-1}$ on decreasing ϵ_g from 1 to 0.8 with $T_g = 290K$. The authors were primarily concerned with the estimating the relative magnitudes of the radiative and turbulent fluxes in the atmospheric surface layer under stable conditions, and did not therefore resolve variation of the gray flux-emissivity on scales as small as those relevant to the

LTM. This brings out another important point. An increasing spatial resolution of the flux-emissivity variation would appear to make the predictions of the gray theory more reliable on smaller length scales. But, at least with the gray formulation given by (2)-(5), an increase in resolution, in serving to resolve length scales characterizing increasingly opaque bands, increases α_{nw} with f_w fixed. Thereby, the disparity between the opacity of the bands involved in the aphysical coupling term increases (see (11)), and the amplitude of the band cross-talk ends up being progressively magnified.

One may ask, based on the above discussion, if the error made in the flux-emissivity models used by Garrat and Brost (1981) ([10]) and VSN (1993) ([2]) is really an intrinsic deficiency of any gray formulation. And, if therefore, radiation problems involving non-black surfaces need to necessarily be handled by band models that more faithfully represent the emission spectrum of the participating medium. The answer is in the negative. The error, in fact, arises in the naive treatment of the reflection term in these gray theories. Again considering (5) for the reflected flux, we note that $F^{\downarrow}(0)$, upon reflection, is attenuated by $(1-\epsilon(u))$, and therefore treated in the same manner as the ground emission (see (2)). Now, it is the spectral distribution of the incident radiation that decides the magnitude and length scale of the attenuation; for instance, in the specific context of a water-vapor-laden atmosphere, an incident radiation with a spectral content entirely restricted to the window band, will pass through an atmosphere of height u_∞ with negligible attenuation. The use of $(1-\epsilon(u))$ for the reflected radiation amounts to re-setting the path-length of the original down-coming radiation to zero; and not recognizing that the emitted radiation has only travelled an upward distance u, while the reflected radiation has, on account of both upward and downward trajectories, interacted with the atmosphere over a greater distance. It is precisely this extended interaction that leads to a difference in the spectral contents of the emitted and reflected radiant fluxes, and makes the treatment in (5) incorrect.

There is, of course, no explicit input of spectral information in a gray formulation, but such information is implicitly contained in $\epsilon(u)$; as already discussed at the beginning of this section, the two-scale nature of the variation in $\epsilon(u)$, given by (6), emphasizes the sharp contrast in the nature of the underlying bands comprising the emission-spectrum. That the actual reflected flux is composed almost entirely of emissions in the opaque bands may therefore be accounted for, even in a gray formulation, if one recognizes that it represents the cumulative downward emission of the atmosphere, and the latter is deficient in the window range. The resolution of the error in (5) thus lies in accounting for the total path-length of the reflected radiation. It is natural then to relate the reflected radiation to the emission of a column of air of height $(u+u_\infty)$ (the total height now being equal to the sum of the distance u travelled upward and the distance u_∞ in the downward direction). The emission of such an isothermal column of air at temperature T_0 is given by $\epsilon(u+u_\infty)\sigma T_0^4$. This is, of course, only the emission, and doesn't take into account the weakening effect of the reflection interaction at the ground. The latter is accomplished by first multiplying the total emission by a factor of $(1-\epsilon_g)$, and then isolating the reflected flux by subtracting the weakened upward emission of the layer of height u that has not interacted with the ground. Thus, the correct expression for the reflected flux contribution to be used in a gray formulation for an isothermal atmosphere must be:

$$F_{rg}^{\uparrow}(u) = (1-\epsilon_g)[\epsilon(u+u_\infty) - \epsilon(u)]\sigma T_0^4, \tag{21}$$

where $(1-\epsilon_g)\epsilon(u)\sigma T_0^4$ denotes the weakened emission of an isothermal layer of height u. Using (21) together with (2)-(4) for the other components of the fluxes, one finds the net radiative flux divergence to be

$$\frac{dF}{du} = \sigma T_0^4[(1-\epsilon_g)\dot{\epsilon}(u+u_\infty) - \dot{\epsilon}(u_\infty - u)], \tag{22}$$

for $K=1$. Further, on using (6) for the gray flux-emissivity, it is readily verified that the (22) yields the same result as the band formulation (see (16)). The error in the perhaps obvious expression for the reflected flux viz. $(1-\epsilon_g)\epsilon(u_\infty)\sigma T_0^4\tau(u)$, lies in the use of the gray flux-transmissivity $\tau(u) = 1 - \epsilon(u)$ for its attenuation. Since the attenuation of any incident radiation depends on its energy distribution among the various bands, an inherent difficulty of any gray formulation is that there is no universal gray transmissivity that can then be used to model the attenuation of any incident radiation; in particular, $\tau(u)$ is the correct attenuation only for an incident radiation emitted by an infinitely high water-vapor-laden atmospheric column that contains the full complement of energy in all its bands. Clearly, $F_a^{\downarrow}(0)$ does not satisfy this latter requirement, and (21) shows that the correct transmissivity to be used for $F_a^{\downarrow}(0)$ is $\tau^*(u) = \frac{\tau(u)-\tau(u+u_\infty)}{1-\tau(u_\infty)}$.

The above argument is easily generalized to obtain the correct formal expression for the reflected flux in an arbitrary inhomogeneous non-isothermal atmosphere:

$$F_{rg}^{\uparrow}(u) = -(1-\epsilon_g)\Big[\int_{u_\infty}^{-u} \sigma T^4(u',t)\dot{\epsilon}(u+u')du' + \int_0^u \sigma T^4(u',t)\dot{\epsilon}(u-u')du'\Big]. \qquad (23)$$

The first term is again related to the cumulative emission of a layer of air of height $(u+u_\infty)$, and the second term removes the weakened upward emission contribution of the air layers between the ground and the layer at u. The latter layers don't interact with the ground, and their emission has already been included in $F_a^{\uparrow}(u)$. The negative sign in the first term arises because the reflected radiation points upward, while the integral itself represents the downward emission of air layers. Note that the original quadratic coupling of emissivities is now absent. Thus, the spurious cooling has been removed even within the gray formulation, and an increasing spatial resolution of the gray flux-emissivity will lead to increasingly accurate predictions for the flux divergence over smaller length scales. Replacing $\sigma T^4(u,t)$ by $B_j(T(u,t))$, the proportion of the total radiant intensity contained in band j, and the gray-flux emissivity, $\epsilon(u)$, by the band-averaged emissivity, $\epsilon_j(u_j)$, leads to the correct expression for the reflected flux to be used in a band formulation:

$$F_{rg}^{\uparrow j}(u_j) = -(1-\epsilon_g)\Big[\int_{u_\infty}^{-u_j} [\pi B_j(T')]\dot{\epsilon}_j(u_j+u_j')du_j' + \int_0^{u_j} [\pi B_j(T')]\dot{\epsilon}_j(u_j-u_j')du_j'\Big]. \quad (24)$$

The above expression eliminates the possibility of cross-talk between sub-bands of varying opacity that led to the error in Varghese et al's band formulation [[28]]. Of course, if one does resolve the spectrum down to scales where the absorption coefficient is a smoothly varying function, that is, down to the sub-band level, there will be no cross-talk. This is easily seen from the earlier expressions for the flux divergence in the gray and band formulations; in the limit $\alpha_w = \alpha_{nw} = \alpha$, both (11) and (16) reduce to the following correct expression for the flux divergence:

$$\frac{dF}{du} = \sigma T_0^4 \Big[-\epsilon_g \alpha \mathrm{e}^{-\alpha u}(K-1) + (1-\epsilon_g)\alpha \mathrm{e}^{-\alpha(u+u_\infty)} + \alpha \mathrm{e}^{-\alpha(u_\infty - u)} \Big] . \tag{25}$$

The above convergence of the gray and band formulations is, of course, no surprise since the medium is indeed gray at the level of resolution.

For purposes of clarity, it is worth re-writing the correct expressions for reflected radiant flux together with the other flux contributions in the gray flux-emissivity formulation

$$\begin{aligned}
F_{eg}^{\uparrow}(u) &= \epsilon_g \sigma T_g^4 (1 - \epsilon(u)), && (26)\\
F_{a}^{\uparrow}(u) &= -\int_0^u \sigma T^4(u', t)\dot{\epsilon}(u - u')du', && (27)\\
F_{a}^{\downarrow}(u) &= \int_{u_\infty}^u \sigma T^4(u', t)\dot{\epsilon}(u' - u)du', && (28)\\
F_{rg}^{\uparrow}(u) &= -(1 - \epsilon_g)\Big[\int_{u_\infty}^{-u} \sigma T^4(u', t)\dot{\epsilon}(u + u')du' + \int_0^u \sigma T^4(u', t)\dot{\epsilon}(u - u')du'\Big] . && (29)
\end{aligned}$$

The above formulation may now be used in situations where the emitting surfaces are not perfect absorbers, and one expects to handle small-scale phenomena accurately with an increasing spatial resolution of the gray flux-emissivity.

3 The origin of the Ramdas layer

Having established that the explanation put forth by the VSN model for the existence of an LTM is wrong, we return to the question of what actually causes the LTM. Rather remarkably, Mukund and Sreenivas (2009) report radiative cooling rates as high as $1 - 5Ksec^{-1}$ in the lowest layer of air of the order of a centimeter above the ground. Even the erroneously amplified near-surface cooling rates predicted by the VSN model are only about $0.02Ksec^{-1}$ ($1700Kday^{-1}$) for $\epsilon_g = 0.8$, and thus, more than an order of magnitude lower than the observations (see figure 4). Now, the net radiative flux in the lowest meter of the atmosphere is about $100W/m^2$. A conductive flux of the same order, as would be the case in a conductive boundary layer, translates to an enormous temperature gradient of about $4000K\,m^{-1}$. Figure 5 shows that the temperature gradient at a height of about one centimetre is already much smaller than the above estimate. Thus, the high cooling rates observed by Mukund and Sreenivas (2009) are not an artifact of conduction, acting in a thin boundary layer just above ground, to resolve any temperature slip induced by long-wave radiation. In fact, the observations suggest that the thickness of such a boundary layer would only be about a millimeter or smaller.

While the prediction of hyper-cooling in the VSN model is an error, the observed hyper-cooling in the near-surface air layers in various experiments ([21];[22];[15]) is real, and needs an explanation. We first examine the possibility that the LTM may simply be a remnant of the day-time temperature profile. As discussed in the introduction, the ground, especially in the tropics, is left considerably hotter than the lower layers of air by the absorption of solar radiation during the day; the temperature gradient at the ground may exceed the nominal adiabatic lapse rate by three or four orders of magnitude. It was therefore suggested by Ramdas (1932) ([21]) that this characteristic of the evening temperature profile might persist through the night even as the ground starts to cool by emission of long-wave radiation. The night-time profile with an LTM is not merely a displaced late-evening temperature profile, however. This is seen in figure 6 which

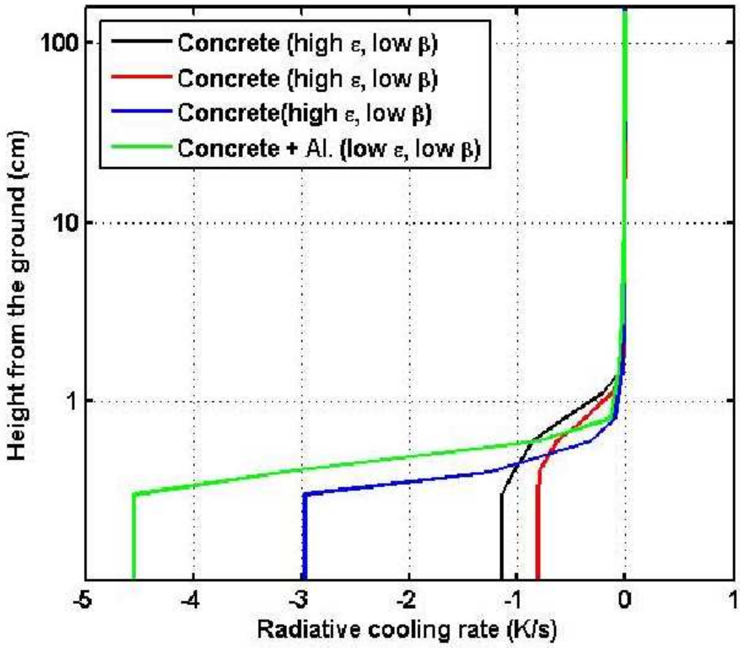

Figure 4: The figure shows the vertical variation of the measured infra-red flux divergences in Mukund and Sreenivas's field experiments ([15]); the flux divergences may also be interpreted as radiative cooling rates.

highlights the qualitative difference between the two profiles. Although the ground is warmer in both cases, the evening temperature profile exhibits a monotonic variation with the hottest layers of air closest to the warmest surface (ground). This is in contrast to the counterintuitive non-monotonic temperature dependence in an LTM profile, already discussed in the introduction.

A couple of other points reinforce the existence of an intrinsic cooling mechanism giving rise to the LTM, rather than it being the result of an imprint left at sunset. Figure 7 shows that air layers, up to a height of $150\,cm$, actually cool *faster* than the ground under calm conditions. The relative magnitudes of the cooling rates are crucial since one has an inherently unsteady scenario with the ground starting to cool at sunset by emission of long-wave radiation, and thereby, 'dragging' the overlying layers of air along with it. Since the ground is warmest at sunset, one would normally expect the near-surface air

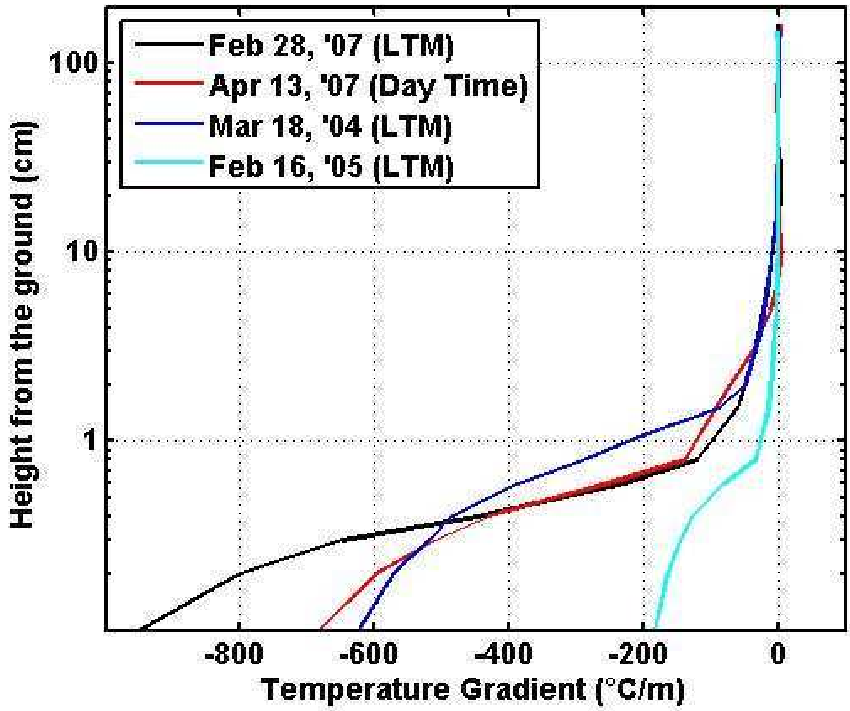

Figure 5: The figure shows the vertical variation of the temperature gradient in the lowest meter of the atmosphere as measured by Mukund and Sreenivas ([15]).

layers to heat up to the ground on account of radiative exchanges in the opaque bands. The photons involved in this exchange typically have mean-free-paths greater than the thin thermal boundary layer in figure 11, and the heating may therefore be interpreted as the familiar one driven by a temperature slip - the term proportional to $(K-1)$ in (11). This radiative heating would lead to a decreasing temperature difference between the air layers and ground even as the respective absolute temperatures continue to decrease after sunset. This is, however, in contrast to the experimental observations. Thus, the cooling of the air-layers relative to the ground can only occur due to radiative exchanges with the upper layers of the atmosphere which are at a temperature substantially lower than either. The slower cooling of the ground itself may be attributed to its higher thermal conductivity and greater thermal inertia ([26]). But, the importance of a 'cooling-to-outer-space' contribution for the air layers implies that the dynamics in the transparent window band must play a crucial role in the underlying radiative balance. Further, that

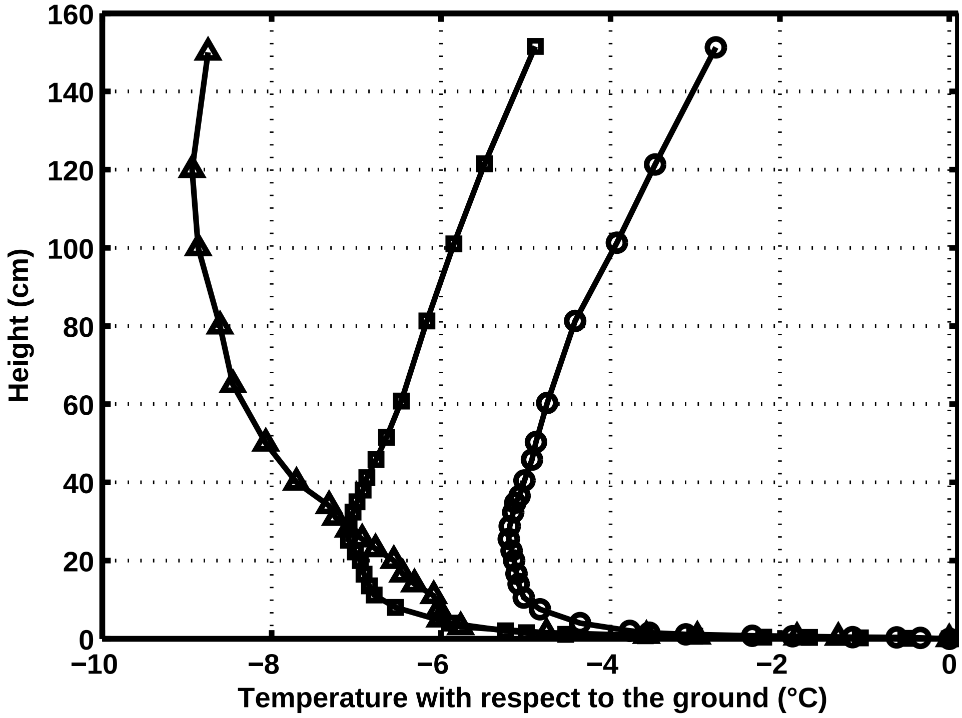

Figure 6: The figure compares a typical day-time temperature profile (triangles) with two typical night-time profiles (squares and circles) exhibiting an LTM in order to highlight the qualitative difference between the two cases; the LTM profiles represent observations by Mukund and Sreenivas (2009).

the radiative exchange involving the layers of air just above the ground and outer space is most efficient can only mean that the constitution of these layers of air must differ from the warmer layers of air above; in other words, an atmosphere that is heterogeneous in the lowest meters. The heterogeneity must then be responsible for enhancing the radiative efficiency, the enhancement made possible by an increased emission in the window band [3]. The existence of a heterogeneity also helps clarify the counter-intuitive nature of the LTM - the existence of cold air in warmer surroundings.

The above arguments assume effects related to advection or turbulent convection to be small. Early observations of the LTM were met with skepticism precisely because of the speculated role of wind in advecting horizontal temperature inhomogeneities. The evidence in figure 7, however, lends unambiguous support to the central role of radiation

[3] Since the heterogeneity may be in the form of microscopic solid particles or water droplets, its emission spectrum, like most solids, is expected to be relatively flat as a function of wavelength; the requirement of an efficient emission in the window band is thus hardly a restrictive one.

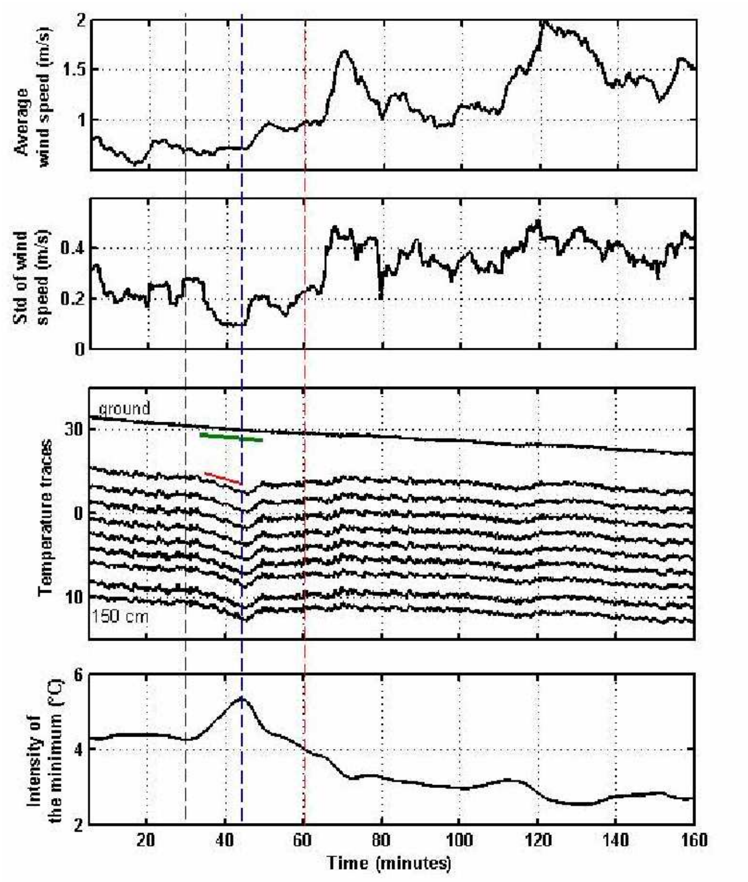

Figure 7: The observed cooling rates of the lowest air layers and that of the ground in relation to the prevailing mean wind speed and fluctuations ([15]). The different temperature traces have been displaced vertically for clarity. The faster cooling of the air layers leads to an increase in the intensity of the LTM when the wind speed drops below a threshold.

in generating the LTM. The observed cooling rates show that the intensity of the minimum $(T_g - T_{min})$ is inversely proportional to both the mean wind speed (characterizing the magnitude of advective effects) and velocity fluctuations (characterizing turbulence transport). In fact, the generation of an LTM, without an apparent violation of the second law of thermodynamics, implies the existence of a non-local heat transfer mechanism. An argument relying solely on local transport mechanisms (conduction and convection) would find it impossible to explain a spontaneous cooling of air layers close to the ground amidst warmer surroundings in a homogeneous atmosphere. Since there is virtually no interaction of these air layers with the upper layers of the atmosphere, the system comprising the near-surface layers of air is closed and the spontaneous emergence of any non-isothermality (due to preferential cooling) would lead to a decrease in entropy. On the other hand, accounting for long-wave radiation, and for the transparency of the heterogeneous atmosphere, renders the system open and capable of radiant energy exchange with cooler outer space.

Finally, a detail concerns the role of the rapid temperature variation within a boundary layer, present in the late-evening temperature profile, in determining the observed cooling rates (see figure 6). The varying temperature itself implies an inhomogeneous atmosphere, and it is known that with such a variation, it is possible for the calculated radiative flux divergence to change sign very close to the ground ([31];[32]). The observed cooling rates even in the air layers just above ground are, however, unrelated to the near-surface isothermality, and this may be seen from the disparate length scales characterizing the observed temperature and flux-divergence profiles (see figures 4 and 6); the latter profile decays over a much smaller characteristic length scale of the order of $1\,cm$. It is worth emphasizing that although the temperature variation very close to the ground may lead to a radiative cooling contribution, it doesn't explain the continued cooling of the air layers above the slip layer. Further, such cooling can never lead to the lowest layers of air attaining a lower temperature than those above them (in accordance with the aforementioned entropy arguments). Thus, the strong non-isothermality of the late-evening atmosphere in the lowest meter is but a detail when it comes to explaining the formation of the LTM, and it makes sense to look for an explanation of the same in the simplified theoretical construct of an initially isothermal atmosphere. It will be argued below that the above radiative mechanism, driven by near-surface temperature variation, nevertheless plays a role in the eventual attainment of radiative equilibrium of the night-time temperature profile exhibiting an LTM.

To sum up, experimental observations clearly show that the origin of the LTM lies in an intrinsic radiative cooling mechanism of the lowest layers of air. However, both with regard to the LTM, and in a more general context, the conclusion that emerges from the analysis in section 2 is that there can exist no preferential radiative cooling or heating mechanism in an atmosphere homogeneous on the relevant length scales. The occurrence of the LTM in particular must therefore be a reflection of an atmosphere that is inhomogeneous on the same length scales as the temperature minimum. The near-surface layers of air, on account of the heterogeneity, a likely efficient emitter in the window band, manage to cool more efficiently to outer space than their immediate surroundings. While the role of a near-surface heterogeneity in the generation of an LTM seems certain, it is difficult to be equally sure as to its exact nature; particularly, in the absence of measurements in the lowest meters of the atmosphere. Thus, in what follows, we present some tentative evidence to suggest the varying concentration of aerosols in the near-surface layers of air as a plausible candidate for the this inhomogeneity. In this regard, the necessary ingredients of a possible theoretical explanation of the LTM are also discussed. Efforts are currently underway to experimentally validate the above hypothesis.

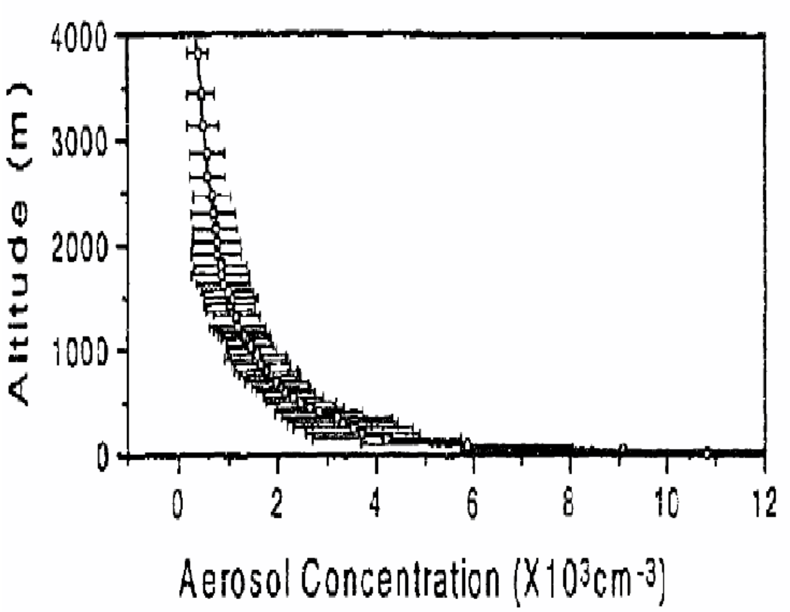

Figure 8: The mean profile of aerosol number density during 17 February-31 March 1998 ([8]).

Figure 8 shows a typical aerosol concentration profile over the lowest kilometers under nocturnal conditions obtained from $LIDAR$ measurements ([8]). There is a clear suggestion of a rapid rise in concentration with approach towards the ground. Unfortunately, the lowest data point is at $50m$; about two orders of magnitude greater than the scales relevant to the LTM! Thus, although the variation in aerosol concentration is consistent with the above hypothesis, it is by no means quantitative. It is nevertheless possible to argue qualitatively, based on the flux-emissivity formulation, for role of a heterogeneity, in the underlying radiation balance. Since the above arguments have already shown that the non-isothermality in the nearest air layers is a relatively minor consideration, we will restrict our attention to an inhomogeneous but isothermal atmosphere. We will only consider the balance within the window band since a near-surface atmospheric inhomogeneity cannot interact with outer space via opaque bands. Denoting the absorption coefficient of a single element of heterogeneity (we shall refer to this element as an aerosol particle from hereon solely for purposes of convenience) as α_h, the transmissivity of an atmospheric column of height u may be written as $\tau(u, \Delta u) = \mathrm{e}^{-\int_u^{u+\Delta u} \alpha(u')du'}$ with $\alpha(u) = \alpha_w + A\, n(u)\alpha_h$, and $\epsilon(u, \Delta u) = 1 - \tau(u, \Delta u)$. Here, α_w is the combined absorption coefficient accounting for the contributions, in the window band, of all atmospheric gases relevant to the lowest meters of the atmosphere; this will be a constant on length scales pertaining to the LTM since the relevant density-scale heights are of the order of kilometers. The second term, $An(u)\alpha_h$, is the appropriately weighted contribution of the aerosols to the transmissivity, where $n(u)$ is the local aerosol number density, and $An(u)$ is a measure of the specific interfacial area (relevant to emission). Note that, in contrast to the homogeneous case, the flux-emissivity in a heterogeneous atmosphere, in addition to depending on the height of the air column (Δu), is also a function of the actual vertical coordinate (u). The above expression for $\alpha(u)$ includes an implicit assumption of diluteness, leading to the aerosol contribution being linearly proportional to the number density. The expected number densities close to the ground do fall in the dilute regime, and any form of radiative interactions between aerosol particles (absorption or multiple scattering) is neglected.

The various contributions to the total radiant flux may again be written in a manner similar to section 2. Since it has already been pointed out that there is no qualitative difference between black and non-black surfaces, we consider only the case $\epsilon_g = 1$ in detail. In this case, $F^{\uparrow}_{rg} = 0$, and the contributions from $F^{\uparrow}_{eg}$ and $F^{\uparrow}_{a}$ cancel. Thus, the only remaining contribution is due to the down-welling flux $F^{\downarrow}_{a}(u)$, and the cooling of the near-surface air layers arises due to the cumulative downward emission of the aerosol particles exceeding the absorbed radiant intensity from outer space. The resulting radiative flux divergence is:

$$\begin{aligned} \frac{dF}{du} &= \frac{dF^{\downarrow}_{a}}{du}, && (30) \\ &= f_w(\sigma T_0^4)\mathrm{e}^{-[\alpha_w(u_\infty - u) + A\alpha_h \int_u^{u_\infty} n(u')du']}[\alpha_w + An(u)\alpha_h], && (31) \end{aligned}$$

and the corresponding cooling rate is given by

$$\frac{\partial T}{\partial t}(u) = -\frac{f_w(\sigma T_0^4)}{\rho C_p}\,\mathrm{e}^{-[\alpha_w(u_\infty - u) + A\alpha_h \int_u^{u_\infty} n(u')du']}[\alpha_w + An(u)\alpha_h]. \qquad (32)$$

The requirement of the aerosols being efficient emitters translates to $An(u)\alpha_h \gg \alpha_w$, so the expression for the magnitude of the cooling rate simplifies to

$$\frac{\partial T}{\partial t}(u) = \frac{f_w(\sigma T_0^4)}{\rho C_p}\,\mathrm{e}^{-A\alpha_h \int_u^{u_\infty} n(u')du'} A\alpha_h n(u). \qquad (33)$$

Since $An(u)\alpha_h \gg \alpha_w$, the cooling rate is much larger in magnitude than that for a homogeneous water-vapor-laden atmosphere. On the other hand, $A\alpha_h \int_0^{u_\infty} n(u')du'$ is still small enough that the entire heterogeneous atmosphere continues to be transparent in the window band. A more intuitive interpretation of this transparency measure may be given in terms of a view-factor, a geometric entity defined as the fraction of the radiation emitted by one surface and arriving at another; view-factors are an integral element of radiative heat transfer calculations involving enclosures ([25]). The restriction $A\alpha_h \int_0^{u_\infty} n(u')du' \ll 1$ essentially amounts to the total emission cross-section of the entire body of suspended aerosol particles being small enough that the view factor of outer space, with respect to ground, is approximately unity. Thus, in the microscopic picture, each aerosol particle near the ground is primarily interacting with outer space in the window band with radiative interactions between different particles being negligible in comparison.

While there continues to be a contribution in the flux divergence, given by (32), that increases with increasing height, the rapidly decaying concentration field leads to an additional contribution that allows for a reversal of the profile for the radiative flux divergence. This reversal would imply that the cooling rate over the lowest meters now peaks at the ground and *decreases* with increasing height, as is required for the formation of an LTM. That such a reversal will occur may be seen from the vertical derivative of the cooling rate given by (33):

$$\frac{\partial}{\partial u}\left(\frac{\partial T}{\partial t}\right) = \frac{f_w(\sigma T_0^4)}{\rho C_p}\, \mathrm{e}^{-A\alpha_h \int_u^{u_\infty} n(u')du'} \left(\{An(u)\}^2\alpha_h^2 + A\alpha_h \frac{dn}{du}\right), \tag{34}$$

where the second term within brackets arises only for a heterogeneous atmosphere. The radiative cooling rate will decrease starting from the ground only if $(\frac{1}{n_0}\frac{dn_0}{du})^{-1} \ll An_0\alpha_h$; here, the subscript '0' denotes quantities evaluated at $u = 0$. The physical interpretation is that, for preferential cooling of the near-surface air layers to occur, the length scale characterizing the aerosol concentration profile must be smaller than the (angularly averaged) path-length in a homogeneous aerosol-laden atmosphere with the number density being equal to its value at the surface (n_0). Thus, the above analysis shows that a sufficiently steep variation of aerosol content will lead to a *decrease* rather than an increase in the radiative cooling rate with height consistent with the intuitive notion of the aerosol-laden lowest layers of air now being able to 'see' outer space better. The associated monotonic temperature profile is sketched in figure 9 (profile a). The above calculation may be repeated for a non-black surface, and the corresponding expression for the flux divergence is given by

$$\frac{dF}{du} = f_w(\sigma T_0^4)An(u)\alpha_h\Big[(1-\epsilon_g)\mathrm{e}^{-A\alpha_h\{\int_0^u n(u')du'+\int_0^{u\infty} n(u')du'\}}+\epsilon_g\mathrm{e}^{-A\alpha_h\int_u^{u\infty} n(u')du'}\Big], \quad (35)$$

with the same assumptions as above. The vertical derivative of the resulting cooling rate (magnitude) is given by

$$\frac{\partial}{\partial u}\left(\frac{\partial T}{\partial t}\right) = \frac{f_w(\sigma T_0^4)}{\rho C_p}\left[(1-\epsilon_g)\mathrm{e}^{-A\alpha_h\{\int_0^u n(u')du'+\int_0^{u\infty} n(u')du'\}}\left(-\{An(u)\}^2\alpha_h^2+A\alpha_h\frac{dn}{du}\right)\right.$$
$$\left.+\,\epsilon_g\mathrm{e}^{-A\alpha_h\int_u^{u\infty} n(u')du'}\left(\{An(u)\}^2\alpha_h^2+A\alpha_h\frac{dn}{du}\right)\right]. \quad (36)$$

Note that in the term proportional to $(1-\epsilon_g)$, both the homogeneous contribution and the one that arises due to the gradient in the aerosol concentration field lead to a cooling rate that decreases with height. This is because the inhomogeneous aerosol distribution leads to an increased attenuation of the deficit in the reflected flux. Thus, one expects higher values for the radiative flux divergence in the near-surface air layers above a reflective surface, again consistent with experimental observations (see figure 4).

The monotonic variation in temperature, with a minimum at the ground, predicted by (34) and (36), is to be expected only at the initial instant. For all later times, there will be radiative heating contributions on account of both the resulting slip between the lowest air layers and ground, and the temperature difference between these layers and the hotter layers of air above. This should lead to a non-monotonic variation of the evolving temperature field (profile b). Eventually, one expects a radiative equilibrium, and therefore, a steady temperature profile, arising from a balance of the three contributions - the heterogeneity-induced radiative cooling and slip-induced radiative heating, in the opaque bands, to both the ground and the upper air layers. It is important to note that, in the present scenario, radiative mechanisms alone are expected to give rise to the non-monotonic temperature dependence characteristic of the LTM. The role of conduction may well be only to smooth any residual slip at the ground over a much shorter length scale (profile c). The distinction made here between the nominal temperature gradient based on the ratio of the intensity and height of the LTM, and the actual temperature gradient in a conductive boundary layer close to the ground, is consistent with recent observations ([15]); as already pointed out, the height of the LTM is of the order of a few decimeters, while the thickness of the conductive boundary layer estimated earlier was of the order of a millimeter (see figure 5).

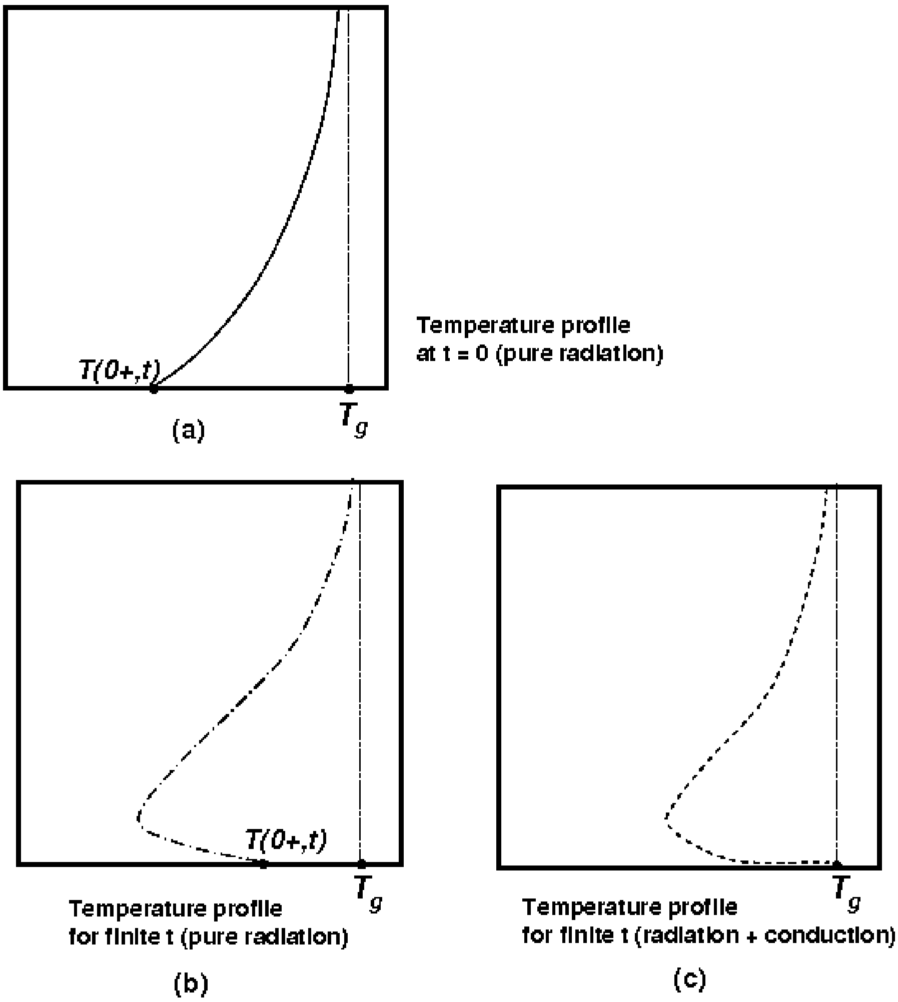

Figure 9: The figure depicts the mechanism of the heterogeneity-induced cooling, leading to the LTM. The first profile is the temperature profile that would result from a heterogeneity-induced cooling, the second includes the combined effects of heterogeneity-induced cooling and slip-heating, while the third also includes the effects of conduction.

The appeal to a heterogeneity-induced cooling also calls for a reinterpretation of what has been recognized as the extreme sensitivity of the LTM to turbulence ([20]). The VSN explanation of the LTM response following a 'gust episode', of course, again relies on the spurious radiative cooling in the opaque bands on a shorter time scale, followed by slow conductive smoothening on a much longer time scale. On the other hand, the gust episode may now be interpreted, at least in part, as a 'homogenization event' restricted to the lowest meters. Thus, even if the turbulent fluxes during a gust were to not dominate the radiative fluxes, the LTM may still disappear simply because of the absence of any strong radiative cooling contribution (flux divergences) in the resulting homogeneous atmosphere. If the gust episode is indeed a homogenization event, then the actual cooling rates of the near-surface air layers after the gust, and the radiative flux divergencecs at these locations, interpreted as cooling rates, reflect entirely different physics. In particular, the former are likely related to the approach of the concentration profile of the heterogeneity towards equilibrium following mixing by turbulence, and are therefore expected to be much lower in magnitude. The observations in figures 4 and 7 are entirely consistent with this interpretation.

The discrepancy between the experimentally measured flux-divergences, and those predicted from radiative models, has, in fact, puzzled researchers for a while. As early as 1960, Funk reported serious disagreement between the measured and theoretically obtained flux divergences, and attributed the discrepancy to 'invisible haze' ([9]). As seen above, this discrepancy is even more pronounced for the case of the LTM. Funk's observations were, in part, the motivation for Zdunkowski's earlier analysis of the effect of atmospheric haze on infrared flux divergences ([32],[30]). In light of the present discussion, credit must be due to the above authors for being among the first to note the necessity of a heterogeneity to explain the LTM. Unfortunately, the advent of the VSN model appears to have misled at least a section of the radiation community into believing that an LTM can occur even in a homogeneous atmosphere. Of course, in the absence of actual measurements of aerosol concentration profiles in the lowest meters, the assumptions underlying the original analysis of Zdunkowski led to the predicted contribution of any haze layer being too small to account for the observed flux divergences, A quantitative prediction of the LTM characteristics, and the underlying infrared flux divergences, therefore remains elusive. For the time being, the Ramdas layer remains a micro-meteorological puzzle, continuing to defy theoretical explanations, close to 80 years after its first documentation.

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