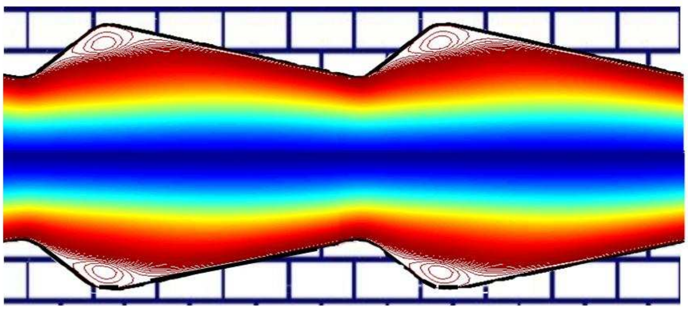
Novel Stability Problems in Pipe Flows

A Thesis
Submitted for the Degree of
Doctor of Philosophy
in the Faculty of Engineering

by

Kirti Chandra Sahu



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To my parents

DECLARATION

*I hereby declare that the matter embodied in the thesis entitled "***Novel Stability Problems in Pipe Flows***" is the result of investigations carried out by me at the Engineering Mechanics Unit, Jawaharlal Nehru Centre for Advanced Scientific Research, Bangalore, India under the supervision of* **Prof. Rama Govindarajan** *and that it has not been submitted elsewhere for the award of any degree or diploma.*

In keeping with the general practice in reporting scientific observations, due acknowledgement has been made whenever the work described is based on the findings of other investigators.

Kirti Chandra Sahu

CERTIFICATE

*I hereby certify that the matter embodied in this thesis entitled "**Novel Stability Problems in Pipe Flows**" has been carried out by **Mr. Kirti Chandra Sahu** at the Engineering Mechanics Unit, Jawaharlal Nehru Centre for Advanced Scientific Research, Bangalore, India under my supervision and that it has not been submitted elsewhere for the award of any degree or diploma.*

Prof. Rama Govindarajan
(***Research Supervisor)***

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Synopsis

The main objective of this thesis is to study the instability and evaluate the likely mechanisms of transition to turbulence in divergent and small-scale (as discussed below) pipe flows. It was first demonstrated by Reynolds (1883) that pipe flow becomes turbulent at a particular value of a non-dimensional parameter which now bears his name. In recent times, this problem has received a lot of renewed attention [Faisst & Eckhardt (2003); Hof *et al.* (2003); Peixinho & Mullin (2006)]. With many demonstrations [see e.g. Hof *et al.* (2004)] that pipe flow may be maintained laminar up to high Reynolds numbers (of the order of hundred thousand), an understanding of the effect of variations in geometry and flow conditions are increasingly relevant.

Fully developed laminar flow (Hagen-Poiseuille flow) through a straight pipe is linearly stable at any Reynolds number, Re. In this case, nonlinear and transient growth mechanisms drive the transition to turbulence. However Hagen-Poiseuille flow is attained only when the pipe wall is straight and smooth and the pipe is long enough. The length required increases with Reynolds number. There are many variations from these conditions where linear instability can play a significant role in transition to turbulence. Some of these situations are addressed in this thesis. They are, flow through (i) a divergent pipe, (ii) a variety of diverging-converging pipes with constant average radius, (iii) diverging pipes/channels with velocity slip at the wall, and (iv) the entry region of a straight pipe. The instabilities of these spatially developing laminar flows is shown to be fundamentally different from flows that do not vary downstream. We have also studied separation in diverging channels and pipes.

It is not in general possible to derive analytical solutions for the laminar flows described above. We obtain the mean flow by solving the steady two-dimensional/axisymmetric Navier-Stokes equations exactly. For the accuracy desired, the computational time required for solving the elliptic equations is very large. A full-multigrid algorithm (FMG) is used to accelerate the convergence. The code is parallelised at the National Aerospace Laboratories, Bangalore. The FMG speeds up the solution by a factor of hundred as compared to many traditional algorithms like Gauss-Seidel and Jacobi iteration technique, and the parallel code (using an eight processor machine) gives a superlinear speed-up of 11 times over a single processor.

(i) The laminar flow through a divergent pipe is shown to be linearly unstable at any angle of divergence a, with the instability critical Reynolds number tending to infinity as the angle of divergence goes to zero. At small a $(< 1^\circ)$ the instability is determined by a parameter $S(x) \equiv a\mathrm{Re}$ describing the basic flow profile, and the mechanism is inviscid. The flow is linearly unstable to the swirl mode for $S > 10$. The instability critical Reynolds numbers are surprisingly low, e.g. about 150 for a divergence of 3°, which would suggest a role for such instabilities in the transition to turbulence.

For small angles of divergence and high Reynolds numbers, an axisymmetric Jeffery-Hamel equation (AJH) is derived to describe the mean flow. At larger angles of divergence (1° or greater) the axisymmetric Navier-Stokes equations are solved directly. The partial differential equations for non-parallel stability are solved as an extended eigenvalue problem by a novel technique. (ii) We then study the effect of local asymmetric convergence/divergence on laminar flow through a pipe of constant *average* radius. The main finding is that the instability behaviour can be changed dramatically by reversing the direction of flow. This is offered as a possible mechanism that could be operating in small-scale flows, due to the presence of wall roughness.

(iii) Fluid dynamics and the role of the walls at small-scale can be very different from that at large scales. We make a minor foray into this regime, by considering the effect of wall slip at Knudsen numbers less than 0.1. Recent studies indicate that a velocity slip at the wall dramatically stabilizes the linear mode in a plane two-dimensional channel, but has very little effect on the algebraic transient growth of disturbances [Lauga & Cossu (2005)]. At microscales, apart from slip, local divergences and convergences of the wall are frequently encountered, here we focus on the effect of divergence. Whereas transient growth is more important in a plane channel, at wall divergences of less than a degree, it is linear instability, taking place two orders of magnitude lower in Reynolds number than in a plane channel, which is dominant. Unlike in a plane channel, the effect of velocity slip at the wall is to reduce stability. Transient growth is shown to be an insignificant player in the process of transition to turbulence.

(iv) The laminar velocity profile through a circular pipe is parabolic once the flow is fully developed. However, the distance l_e required to reach this fully-developed state can be very long, and scales linearly with the Reynolds number, Re, roughly as $l_e/R \sim \mathrm{Re}/20$, where R is the pipe radius. Therefore high Reynolds number laminar flow through a pipe of limited length may never reach a parabolic state. We show that in such circumstances linear stability can play an important role in transition. We solve for the basic flow exactly, and conduct a non-parallel stability analysis, to show that flow can be linearly unstable even at a Reynolds number of 1000. In contrast to what is expected of a boundary layer type flow, disturbance growth is higher in the core region. Our results are consistent with that of experiment. Earlier theoretical studies predicted critical Reynolds number an order of magnitude higher than that observed in experiment and were in serious disagreement with each other.

(v) Some other studies which are in the preliminary stage are described at the end of the thesis. These include the study of pulsatile flow through a straight pipe. Prescribing time periodic velocity profiles at the inlet we have solved the Navier-Stokes equation directly using a full-multigrid algorithm on a parallel machine Venkatesh *et al.* (2005). We also studied separated flows in the diverging channel/pipe. At higher angles of divergence, we study flow separation in channels and pipes. The size, location and shape of the separated region for divergent angle varying from 0 to 90 degree are discussed in the thesis. In future, we are planning to conduct full non-parallel stability analysis e.g. global stability analysis of such flows without the approximations made in this thesis.

Refereed Publications

1. “Stability of flow through a slowly diverging pipe”, Kirti Chandra Sahu and Rama Govindarajan, *J. Fluid Mech.*, 531, 2005, 325 - 334.

2. “Super-linear speed-up of a parallel multigrid Navier-Stokes solver on Flosolver”, T. N. Venkatesh, V. R. Sarasamma, Rajalakshmy S., Kirti Chandra Sahu and Rama Govindarajan, *Current Science*, 88(4), 2005, 589 - 593.

3. “A possible linear instability mechanism in small-scale pipe flows”, Kirti Chandra Sahu, *Proceedings of Sixth IUTAM Symposium on Laminar-Turbulent Transition*, Springer, 2006, 127-132.

4. “Instability of entry flow in a pipe”, Kirti Chandra Sahu and Rama Govindarajan, *Proceedings of Eleventh Asian Congress of Fluid Mechanics*, May 22-25, 2006, Kuala Lumpur, Malaysia.

5. “Linear instability of the developing flow in the entry region of a pipe”, Kirti Chandra Sahu and Rama Govindarajan, *ASME Journal of Fluids Engineering*, to appear

Conference Paper

6. “Instabilities in decelerating pipe flow”, *Proceedings of STATPHYS 22*, Bangalore, July, 2004.

Preprints

7. “The relative roles of linear instability and transient growth in divergent channel flow with slip”, Kirti Chandra Sahu, A. Sameen and Rama Govindarajan, preprint

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