Orientation Dynamics of Anisotropic Particles in Viscoelastic Fluids

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DECLARATION

I hereby declare that the matter embodied in the thesis entitled "**Orientation Dynamics of Anisotropic Particles in Viscoelastic Fluids**" is the result of investigations carried out by me at the Engineering Mechanics Unit, Jawaharlal Nehru Centre for Advanced Scientific Research, Bangalore, India under the supervision of Dr. Ganesh Subramanian and that it has not been submitted elsewhere for the award of any degree or diploma.

In keeping with the general practice in reporting scientific observations, due acknowledgment has been made whenever the work described is based on the findings of other investigators.

Vivekanand Dabade

CERTIFICATE

I hereby certify that the matter embodied in this thesis entitled "**Orientation Dynamics of Anisotropic Particles in Viscoelastic Fluids**" has been carried out by Mr. Vivekanand Dabade at the Engineering Mechanics Unit, Jawaharlal Nehru Centre for Advanced Scientific Research, Bangalore, India under my supervision and that it has not been submitted elsewhere for the award of any degree or diploma.

Dr. Ganesh Subramanian
(Research Supervisor)

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Contents

Abstract

The present work concerns the orientation dynamics of anisotropic particles in viscoelastic fluids. A spheroidal geometry is taken as being representative of an anisotropic, axisymmetric particle, and the work is an analytical investigation of the effects of weak inertia and viscoelasticity on the orientation of a spheroidal particle (both prolate and oblate) of an arbitrary aspect ratio in two canonical flow situations:

1. A spheroid sedimenting in a quiescent fluid, and
2. A neutrally buoyant spheroid in a simple shear flow

Applications include sedimentation of muds and slurries, processing of cellulose fiber suspensions in the paper and pulp industry, and processing of filled polymeric materials (wherein anisotropic clay particles are typically used as cheap filler materials).

In the absence of both inertia and viscoelasticity, the orientation dynamics of a spheroid is governed by the Stokes equations. On account of reversibility, a sedimenting spheroid continues to retain its initial orientation, while a neutrally buoyant spheroid in simple shear continues to rotate in an initially chosen (Jeffery) orbit. In either case, the particle orientation distribution remains indeterminate as a result. In situations where the characteristics of the motion of a single anisotropic particle may be applied to the calculation of a bulk property of a dilute non-interacting suspension of such particles, the aforementioned indeterminacy in the orientation distribution presents an impediment. In order to eliminate the indeterminacy, and thereby, arrive at a unique orientation distribution, it becomes necessary to consider the influence of additional physical phenomena. Possible candidates for the resolution of the indeterminacy include Brownian motion, pair-particle hydrodynamic interactions, fluid inertia, viscoelasticity of the suspending fluid, etc.

The present work focuses on weak fluid inertia and viscoelasticity with a weakly

viscoelastic fluid being modeled as a second-order fluid in the analysis. It is important to note here that even a weak deviation from the Stokes limit will invariably have a strong cumulative effect over long times precisely due to the above indeterminacy. For instance, a weak inertial torque, acting on a sedimenting spheroid, stabilizes orientations transverse to gravity, while a weak viscoelastic torque ends up stabilizing the longitudinal (vertical) orientation. In both cases, the indeterminacy in the orientation distribution is eliminated. One anticipates a similar situation for the case of simple shear flow.

The canonical motions of a spheroid referred to above, sedimentation in particular, are classic problems, and have been extensively investigated, both theoretically and experimentally, by various authors. The present work analyzes both problems using a new approach based on the formalism of vectorial spheroidal harmonics. The formalism was developed by Vladimir Kushch [see Kushch & Sangani (2003)] and has a structure similar to the well-known spherical harmonics formalism which owes its origin to Lamb (1932). Unlike earlier approaches, and in a manner similar to spherical harmonics, the spheroidal harmonics formalism is readily extended to a multi-particle scenario wherein hydrodynamic interactions between anisotropic particles may begin to play an important role in determining the orientation dynamics. However, as a first step, in this work, the formalism is applied to the motion of a single particle, and the results obtained compared to those of earlier investigations.

The formalism, together with the use of the generalized reciprocal theorem, is first applied to the sedimentation problem, leading to closed-form analytical expressions for the $O(Re)$ inertial and $O(De)$ viscoelastic torques in both sedimentation and simple shear flow as a function of the spheroid aspect ratio. Here, the Reynolds number and the Deborah number denote the scaled magnitudes of the inertial and viscoelastic torques, respectively. Since the two torques act in opposite directions, a balance of the two leads to a neutral curve, that is, a critical value of De/Re as a function of the particle aspect ratio [see figures 3.13 & 3.15] that separates regions where transverse and longitudinal orientations are stable. Despite extensive work on this classic problem, our fully analytical approach shows some of the earlier results to be incorrect. In particular, it is shown that the viscoelastic torque always tends to zero in the limit of an infinitely slender particle for an arbitrary ratio of the two normal stress differences [see §3.2.1]. In fact, the viscoelastic torque becomes extremely sensitive to changes in the particle aspect ratio in the limit of very slender particles, and this may be one reason why earlier numerical calculations of the same turn out to be erroneous [see §3.2.1].

The inertial and viscoelastic contributions to the angular velocity of a neutrally buoyant spheroid in simple shear flow are currently being calculated. The simple shear flow problem is inherently more complicated, since the orientation of the spheroid now changes as a function of time even in the inertialess limit (as the particle moves along a Jeffery orbit) [see Jeffery (1922)]. Further, unlike sedimentation where the only stable orientations turn out to the transverse and longitudinal ones, the more complicated angular dependencies in simple shear flow allow, in principle, for the existence of stable intermediate orbits (that is, in between the limits of in-plane tumbling and log-rolling). Significant progress has already been made, and in the near future, we expect to be able to map out the orientation dynamics in presence of the (possibly) competing effects of viscoelasticity and inertia. Our aim then is to obtain an orbit-constant-surface as a function of De/Re and the particle aspect ratio [see figure 4.1].